Question1 20 marks Similar to QMquiz in courseworks

version ①

Question 1 a) to d) are single choice questions

a) Which of the following ordinary differential equations (ODEs) has the independent variable as t and can be solved by the separation of variables method? (4 marks)

- ( ) I and IV
- (•) I and II
- ( ) II and III,

where I: $\sin y \frac{dy}{dt} = t + 1$, II: $\dot{y} = y^2 e^t$, III: $y' = 2xy$, IV: $\dot{y} = \cos y + t$.

b) Which of the following ODEs can be reduced to be separable? (4 marks)

- ( ) I
- (•) I,II and III
- ( ) II and III
- ( ) None of the ODEs,

where I: $y' = 2x + 3y$, II: $y' = \frac{y}{x} + 1$, III: $y' = \cos(y + x + 2)$.

c) Which of the following statements is correct for solving the ODE, $y' = y + 2x + 1$? (4 marks)

- ( ) The ODE can be reduced to be separable;
- ( ) It is an inhomogeneous linear ODE and can be solved by the variation of parameter method;
- ( ) The method to solve exact ODEs can be applied;
- (•) It can be solved by reducing to be separable or by the variation of parameter method.

d) The solution to the initial value problem $y' = y + 2x + 1$, $y(0) = 5$ is, (8 marks)

- ( ) I
- ( ) II
- (•) III
- ( ) III and IV

where
I: $y(x) = -3 - 2x + Ce^x$, where C is an arbitrary constant,
II: $y(x) = C + x + e^x$, where C is an arbitrary constant,
III: $y(x) = -3 + 8e^x - 2x$,
IV: $y(x) = 4 + x + e^x$.

②

Question 1 a) to d) are single choice questions

a) Which of the following ordinary differential equations (ODEs) has the independent variable as t and can be solved by the separation of variables method? (4 marks)

- ( ) I and IV
- ( ) II and III,
- (•) I and III

where

I: $\dot{y} = y^3 e^t$, II: $y' = 4xy$, III: $y^2 \frac{dy}{dt} = \sin t + 1$, IV: $\dot{y} = \sin y + t$.

b) Which of the following ODEs can be reduced to be separable? (4 marks)

- ( ) I
- ( ) I and II
- (•) II and III
- ( ) None of the ODEs,

where I: $y' = x^2 y + 2y^2$, II: $y' = \frac{3y}{x} + 2$, III: $y' = \cos(y + x + 2)$.

c) Which of the following statements is correct for solving the ODE, $y' = 3y + 2x + 1$? (4 marks)

- ( ) The method to solve exact ODEs can be applied;
- (•) It can be solved by reducing to be separable or by the variation of parameter method.
- ( ) It is an inhomogeneous linear ODE and can be solved by the variation of parameter method;
- ( ) The ODE can be reduced to be separable;

d) The solution to the initial value problem $y' = 4y + x + 2$, $y(0) = 0$ is, (8 marks)

- (•) I
- ( ) II
- ( ) III
- ( ) IV

where
I: $y(x) = -\frac{9}{16} - \frac{x}{4} + \frac{9}{16}e^{4x}$,
II: $y(x) = C - \frac{x}{4} + e^{4x}$, where C is an arbitrary constant,
III: $y(x) = -\frac{9}{16} - \frac{x}{4} + Ce^{4x}$, where C is an arbitrary constant,
IV: $y(x) = -1 - \frac{x}{4} + e^{4x}$.

③

Question 1 a) to d) are single choice questions

a) Which of the following ordinary differential equations (ODEs) has the independent variable as t and can be solved by the separation of variables method? (4 marks)

- (•) I and IV
- ( ) I and II,
- ( ) II and III

where

I: $\sqrt{y}\frac{dy}{dt} = t^2 + 5$, II: $y' = (3x + 1)y$, III: $\dot{y} = \sin y + t$, IV: $\dot{y} = \sqrt{y}e^t$.

b) Which of the following ODEs can be reduced to be separable? (4 marks)

- ( ) I
- (•) II and III
- ( ) I and III
- ( ) None of the ODEs,

where I: $y' = y^2 + 2x^2 y$, II: $y' = 2(\frac{y}{x} + 1)$, III: $y' = \sin(y + 2x + 4)$.

c) Which of the following statements is correct for solving the ODE, $y' = 2y + 3x$? (4 marks)

- (•) It can be solved by reducing to be separable or by the variation of parameter method.
- ( ) The method to solve exact ODEs can be applied;
- ( ) It is an inhomogeneous linear ODE and can be solved by the variation of parameter method;
- ( ) The ODE can be reduced to be separable;

d) The solution to the initial value problem $y' = 2y + 3x$, $y(0) = 0$ is, (8 marks)

- ( ) I
- ( ) II
- ( ) III
- (•) IV

where
I: $y(x) = C - \frac{3x}{2} + e^{2x}$, where C is an arbitrary constant,
II: $y(x) = -1 - \frac{3x}{2} + e^{4x}$,
III: $y(x) = -\frac{3}{4} - \frac{3x}{2} + Ce^{2x}$, where C is an arbitrary constant,
IV: $y(x) = -\frac{3}{4} - \frac{3x}{2} + \frac{3}{4}e^{2x}$.

1 Question two: 10 marks similar to QMquiz Courseworks

Find the right match for the following ODEs in the dropdown menu

$y' = \frac{x}{y+1}$	1st-order nonlinear ODE
$xy' = 2x^2y'' + 3y$	Euler-type ODE
$y' + 2y'' = \sin x\, y + 2$	2nd-order linear inhomogeneous ODE
$y' = 1 + 2xy$	1st-order linear ODE
$y'' = \frac{2x}{y}$	None of the above forms

2

Find the right match for the following ODEs in the dropdown menu

$4y'' = \cos x\, y + \sin x$	2nd-order linear inhomogeneous ODE
$y' = 2x^2y$	1st-order linear ODE
$y' = \frac{x}{2y+3}$	1st-order nonlinear ODE
$y'' = \frac{2x}{y}$	None of the above forms
$5x^2y'' = 3xy' + y$	Euler-type ODE

3

Find the right match for the following ODEs in the dropdown menu

$y'' = \frac{x}{y}$	None of the above forms
$2y'' = y + \sin x\, y' + x^3$	2nd-order linear inhomogeneous ODE
$y' = \sqrt{x}y + x$	1st-order linear ODE
$y' = \frac{2x}{y}$	Scale-invariant ODE
$4x^2y'' + xy' = 7y$	Euler-type ODE

Question 3 10 marks

Similar to QMQuiz in courseworks

1

Question 3 a) to c) are single choice questions.

a) The general solution of the homogenous ODE, $y' = \frac{y}{x}$ is (3 marks)

- (selected) $y(x) = Cx$,
- $y(x) = C + x$,
- $y(x) = x$,

where C is an arbitrary constant.

b) The general solution to the ODE, $y' = \frac{y}{x} + 2x$, can be solved by (2 marks)

- the exact ODE method.
- the separation of variables method.
- (selected) the variation of parameter method.
- none of the methods list.

The general solution to this ODE is (2 marks)

- $y(x) = 2x^2 + x$,
- $y(x) = Cx^2 + 2x$,
- (selected) $y(x) = Cx + 2x^2$,
- $y(x) = 2x + Cx$,

where C is an arbitrary constant.

c) For the initial value problem, $y' = \frac{y}{x} + 2x, y(0) = 0$, it has (3 marks)

- a unique solution, as we can fixed the arbitrary constant by using the initial condition.
- a unique solution, and it fulfils the Picard-Lindelof Theorem conditions.
- (selected) no unique solution, and it does not fulfil the Picard-Lindelof Theorem conditions.
- two solutions, as there are two functions y(x) passing the initial condition in the xy plane.

2

Question 3 a) to c) are single choice questions.

a) The general solution of the homogenous ODE, $y' = \frac{2y}{x}$ is (3 marks)

- $y(x) = C + x^2$,
- (selected) $y(x) = Cx^2$,
- $y(x) = x^2$,

where C is an arbitrary constant.

b) The general solution to the ODE, $y' = \frac{2y}{x} + 2x^2$, can be solved by (2 marks)

- (selected) the variation of parameter method.
- the separation of variables method.
- the exact ODE method.
- none of the methods list.

The general solution to this ODE is (2 marks)

- $y(x) = x^2 + x^3$,
- (selected) $y(x) = Cx^2 + 2x^3$,
- $y(x) = Cx^2 + 2x^2$,
- $y(x) = x^2 + Cx^3$,

where C is an arbitrary constant.

c) For the initial value problem, $y' = \frac{2y}{x} + 2x^2, y(0) = 0$, it has (3 marks)

- a unique solution, as we can fixed the arbitrary constant by using the initial condition.
- two solutions, as there are two functions y(x) passing the initial condition in the xy plane.
- a unique solution, and it fulfils the Picard-Lindelof Theorem conditions.
- (selected) no unique solution, and it does not fulfil the Picard-Lindelof Theorem conditions.

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Question 4 6 marks Options are randomly shuffled

What are the common things in using the variation of parameter methods to solve 1st-order and 2nd-order linear inhomogeneous ODEs?

Select one or more:

- [x] We need to vary the arbitrary constant(s) in the general solution to the homogenous ODE to an unknown function of x.
- [x] We need to differentiate the assumed solution until the order of the ODE, i.e. getting the expression of y' (or y' and y"), and put it (them) back to the original ODE.
- [ ] We need to vary the arbitrary constant(s) in the general solution to the homogenous ODE to an unknown function of y.
- [x] We need to solve the homogenous ODE first.
- [ ] The coefficients in front of the dependent variable and its derivatives need to be constant.

Question 5 (14 marks) Similar to QMquiz in courseworks

Question 5 a) to c) are single choice questions.

a) The general solution of the ODE, $3y'' + 2y' - y = 0$ is (5 marks)

○ I ○ II ◉ III ○ IV ○ None of the solutions listed below,

where

I: $y(x) = C_1 \sin(\frac{x}{3}) + C_2 \cos x$,
II: $y(x) = C_1 e^{-x} + C_2 e^{3x}$,
III: $y(x) = C_1 e^{\frac{x}{3}} + C_2 e^{-x}$,
IV: $y(x) = C_1 x + C_2 x^{-3}$,
and C_1 and C_2 are arbitrary constants.

b) The general solution to the ODE, $3y'' + 2y' - y = e^{-x}$, can be solved by (3 marks)

○ both education guess method and the variation of parameter method. ○ the educated guess method. ◉ the variation of parameter method. ○ none of the methods list.

c) The general solution to $3y'' + 2y' - y = e^{-x}$ is (6 marks)

○ I ◉ II ○ III ○ IV ○ None of the solutions listed below,

where I: $y(x) = (C_1 - \frac{x}{4})e^{x} + C_2 e^{-\frac{x}{3}}$,
II: $y(x) = (C_1 - \frac{x}{4})e^{-x} + C_2 e^{\frac{x}{3}}$,
III: $y(x) = (C_1 - \frac{x}{4})\sin(\frac{x}{3}) + C_2 \cos x$,
IV: $y(x) = C_1 e^{\frac{x}{3}} - \frac{x}{4}e^{-x} + C_2$,
and C_1 and C_2 are arbitrary constants.

2

Question 5 a) to c) are single choice questions.

a) The general solution of the ODE, $3y'' + 2y' - y = 0$ is (5 marks)

○ I ○ II ◉ III ○ IV ○ None of the solutions listed below,

where

I: $y(x) = C_1 \sin(\frac{x}{3}) + C_2 \cos x$,
II: $y(x) = C_1 e^{-x} + C_2 e^{3x}$,
III: $y(x) = C_1 e^{\frac{x}{3}} + C_2 e^{-x}$,
IV: $y(x) = C_1 x + C_2 x^{-3}$,
and C_1 and C_2 are arbitrary constants.

b) The general solution to the ODE, $3y'' + 2y' - y = e^{-x}$, can be solved by (3 marks)

○ both education guess method and the variation of parameter method. ○ the educated guess method. ◉ the variation of parameter method. ○ none of the methods list.

c) The general solution to $3y'' + 2y' - y = e^{-x}$ is (6 marks)

○ I ◉ II ○ III ○ IV ○ None of the solutions listed below,

where I: $y(x) = (C_1 - \frac{x}{4})e^{x} + C_2 e^{-\frac{x}{3}}$,
II: $y(x) = (C_1 - \frac{x}{4})e^{-x} + C_2 e^{\frac{x}{3}}$,
III: $y(x) = (C_1 - \frac{x}{4})\sin(\frac{x}{3}) + C_2 \cos x$,
IV: $y(x) = C_1 e^{\frac{x}{3}} - \frac{x}{4} e^{-x} + C_2$,
and C_1 and C_2 are arbitrary constants.

3

Question 5 a) to c) are single choice questions.

a) The general solution of the ODE, $4y'' + 2y' + y = 0$ is (5 marks)

○ I ○ I and II ○ III ○ IV ○ None of the solutions listed below,

where I: $y(x) = C_1 \cos(\frac{\sqrt{3}x}{4}) + C_2 \sin(\frac{\sqrt{3}x}{4})$,
II: $y(x) = C_1 e^{-\frac{x}{4}} + C_2 e^{\frac{\sqrt{3}x}{4}}$,
III: $y(x) = e^{\frac{\sqrt{3}x}{4}}(C_1 \cos(-\frac{x}{4}) + C_2 \sin(-\frac{x}{4}))$,
IV: $y(x) = e^{-\frac{x}{4}}(C_1 \cos(\frac{\sqrt{3}x}{4}) + C_2 \sin(\frac{\sqrt{3}x}{4}))$,
and C_1 and C_2 are arbitrary constants.

b) The general solution to the ODE, $4y'' + 2y' + y = e^{-x}$, can be solved by (3 marks)

○ both education guess method and the variation of parameter method. ○ the educated guess method. ○ the variation of parameter method. ○ none of the methods list.

c) The general solution to $4y'' + 2y' + y = e^{-x}$ is (6 marks)

○ I ○ II ○ III ○ IV ○ None of the solutions listed below,

where I: $y(x) = C_1 e^{\frac{\sqrt{3}x}{4}}(\cos(-\frac{x}{4}) + \sin(-\frac{x}{4})) + C_2 e^{2x}$,
II: $y(x) = C_1 e^{-\frac{x}{4}} + C_2 e^{\frac{\sqrt{3}x}{4}} + e^{2x}$,
III: $y(x) = e^{-\frac{x}{4}}(C_1 \cos(\frac{\sqrt{3}x}{4}) + C_2 \sin(\frac{\sqrt{3}x}{4})) + \frac{1}{3}e^{-x}$,
IV: $y(x) = C_1 \cos(\frac{\sqrt{3}x}{4}) + C_2 \sin(\frac{\sqrt{3}x}{4}) + e^{-x}$,
and C_1 and C_2 are arbitrary constants.

Question 6. (10 marks)

Version 1   Similar to QM quiz in course works

Question 6 a) to b) are single choice questions.

a) Which of the following problems is a boundary value problem (BVP) with linear boundary conditions? (4 marks)

- I
- I and II
- I, II and III
- None of the problems listed below,

where I: $y'' + y - 5 = 0, y'(\pi) = -1, y(0) = 1,$
II: $y'' + y - 5 = 0, y(\pi) = 9, y'(0) + y(0) = 2,$
III: $y'' + y - 5 = 0, y(0) = 1, y'(0) = 1.$

b) Which of the following statements is correct? (6 marks)

- I
- II
- I, II and III
- II and V
- I, II and V
- I, II and IV
- All of the statements below
- None of the statements below

where I: All problems in 6a) have the same solution.
II: All BVPs in 6a) are inhomogeneous.
III: In order to check whether the BVPs have a unique solution, we need to solve the BVPs in 6a) first and find out the values of arbitrary constants in the general solution by the fulfilling boundary conditions.
IV: All BVP in 6a) have the same corresponding homogeneous BVP.
V: According to the Theorem of Alternatives, all BVPs in 6a) have a unique solution.

2

Question 6 a) to b) are single choice questions.

a) Which of the following problems is a boundary value problem (BVP) with linear boundary conditions? (4 marks)

- I
- II and III
- I, II and III
- None of the problems listed below,

where I: $y'' + 16y = 0, y(\pi) = -2, y'(0) = 2,$
II: $y'' + 16y = 0, y(\pi) = -2, y'(0) + y(0) = 4,$
III: $y'' + 16y = 0, y(\pi) + y'(\pi) = 0, y'(0) = 2.$

b) Which of the following statements is correct? (6 marks)

- I
- II
- II and V
- I, II and III
- I, II and V
- I, II and IV
- All of the statements below
- None of the statements below

where I: All BVPs in 6a) are inhomogeneous.
II: According to the Theorem of Alternatives, all BVPs in 6a) have a unique solution.
III: All BVPs in 6a) have the same corresponding homogeneous BVP.
IV: All BVPs in 6a) have the same solution.
V: In order to check whether the BVPs have a unique solution, we need to solve the BVPs in 6a) first and find out the values of arbitrary constants in the general solution by the fulfilling boundary conditions.

(3)

Question 6 a) to b) are single choice questions.

a) Which of the following problems is a boundary value problem (BVP) with linear boundary conditions? (4 marks)

- ○ I
- ○ II and III
- ◉ I, II and III
- ○ None of the problems listed below,

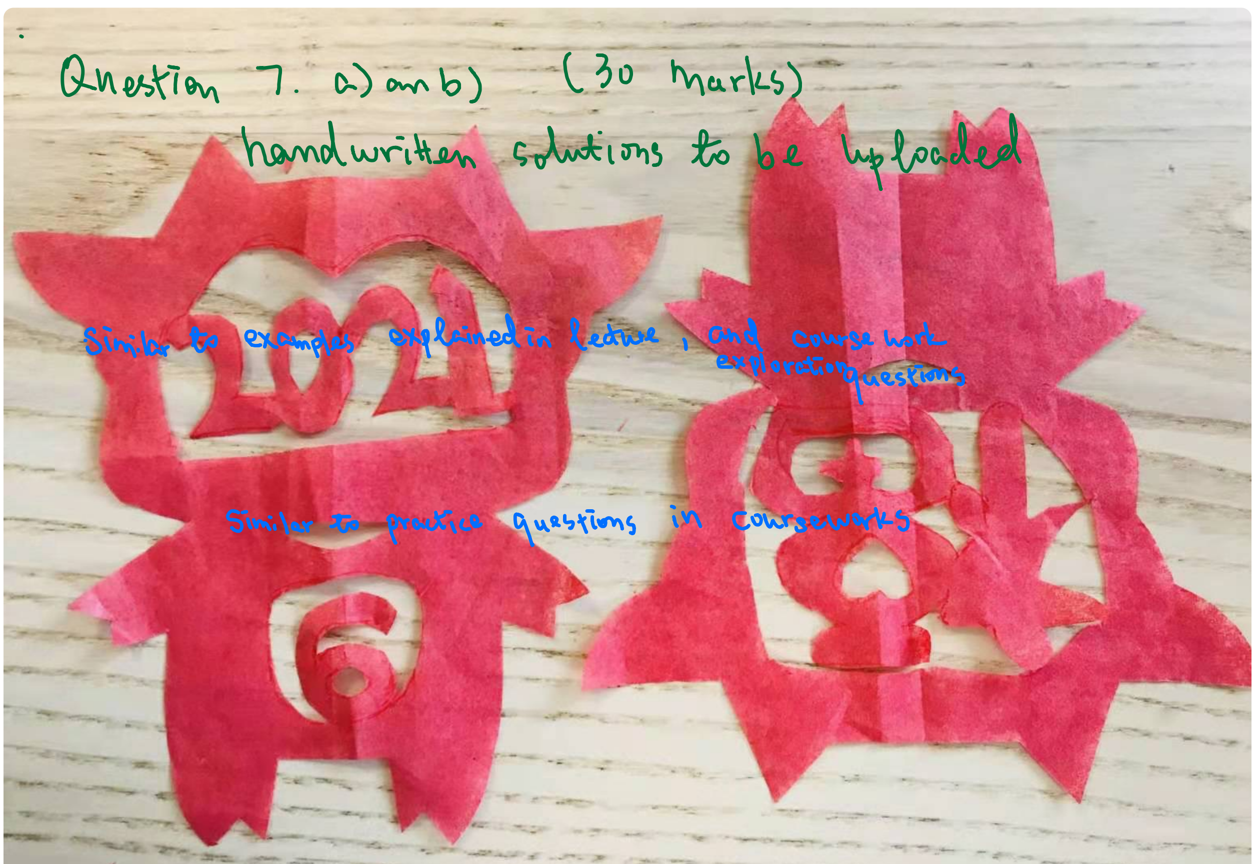
where I: $4y'' + 25y = 0, y(0) = -1, y'(2\pi) = -5,$
II: $4y'' + 25y = 0, y(2\pi) = 1, y'(0) + y(0) = 4,$
III: $4y'' + 25y = 0, y(2\pi) + y'(2\pi) = -4, y'(0) = 5.$

b) Which of the following statements is correct? (6 marks)

- ○ I
- ○ II
- ○ II and IV
- ◉ I, II and III
- ○ II, III and IV
- ○ All of the statements below
- ○ None of the statements below

where I: All BVPs in 6a) are inhomogeneous.
II: According to the Theorem of Alternatives, all BVPs in 6a) have a unique solution.
III: All BVPs in 6a) have the same solution.
IV: All BVPs in 6a) have the same corresponding homogeneous BVP.
V: In order to check whether the BVPs have a unique solution, we need to solve the BVPs in 6a) first and find out the values of arbitrary constants in the general solution by the fulfilling boundary conditions.

Question 7. a) an b) (30 marks)
handwritten solutions to be uploaded



Similar to examples explained in lecture, and course work exploration questions

Similar to practice questions in courseworks

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Image (PNG) .png
PDF document .pdf

Question7:

1. **Question 7**

   Question 7 should be solved by hand in paper, and solutions should be uploaded here with a singled combined pdf.

   a) In a predator-prey system, the number of preys $P(t)$ and predators $K(t)$ will change over time t according to the following ODE system,

   $$\dot{P} = rP - dPK, \quad \dot{K} = gPK - \delta K,$$

   where r, d, g and δ are parameters not variables. Here, r is the growth rate of the preys $P(t)$, d is the death rate of the preys due to predation, g is the growth rate of the predator due to predation and δ is the death rate of the predator $K(t)$.

   Identify the dependent and independent variables in this ODE system. Compute all equilibria of this ODE system and explain the meaning of these equilibria. (10 marks)

   b) Linearise the ODE system,

   $$\dot{y_1} = (y_1 y_2 - 2)y_2, \quad \dot{y_2} = y_1 - y_2.$$

   around the equilibrium when $y_1 = 0$. Calculate the corresponding eigenvalues and eigenvectors of this linearised system. Determine the type of this equilibrium when $y_1 = 0$ (stable node sink, unstable node source,saddle, center, stable spiral, or unstable spiral) and sketch its phase portrait. (20 marks)

10

Sol 7 a) • The dependent variables are P and K. The independent variable is t. 2

• Equilibria are under

$$\begin{cases} \dot{P}=0 \\ \dot{K}=0 \end{cases}$$

$$\Rightarrow \begin{cases} rP-dPK=0 \Rightarrow \begin{cases} P=0 \\ K=\frac{r}{d} \end{cases} \\ gPK-\delta K=0 \Rightarrow \begin{cases} K=0 \\ P=\frac{\delta}{g} \end{cases} \end{cases}$$

4

Thus to fulfill $\dot{P}=0$, $\dot{K}=0$ we have two equilibria

$$\begin{cases} P=0 \\ K=0 \end{cases} \quad \begin{cases} P=\frac{\delta}{g} \\ K=\frac{r}{d} \end{cases}$$

• The first equilibria means both the prey and predator species go extinct. 1

• The second equilibria means the two species coexist, with the 3 prey abundance at $\frac{\delta}{g}$, the predator abundance at $\frac{r}{d}$. Thus, when the preys grow fast (large r), and die slow (small d), more predators will be maintained under this equilibrium. Similarily, when the predators die fast (large δ) or the predators grow slow (small g), more prey will be maintained under equilibrium.

Exam Solution 7b

b) Consider a system of two first-order ODEs,

$$\dot{y_1} = (y_1y_2 - 2)y_2, \quad \dot{y_2} = y_1 - y_2.$$

Linearise the above ODE system around the equilibrium when $y_1 = 0$. Calculate the corresponding eigenvalues and eigenvectors of this linearised system. Determine the type of this equilibrium when $y_1 = 0$ (stable node sink, unstable node source,saddle, center, stable spiral, or unstable spiral) and sketch its phase portrait. (20 marks)

Solution. The equilibrium with $y_1 = 0$ is at $(0, 0)$ (1 mark). To linearize around this point, we need to evaluate $\frac{\partial f_1}{\partial y_1}, \frac{\partial f_1}{\partial y_2}, \frac{\partial f_2}{\partial y_1}, \frac{\partial y_2}{\partial y_2}$ at the point of equilibrium, where $f_1 = (y_1y_2 - 2)y_2$ and $f_2 = y_1 - y_2$. We obtain

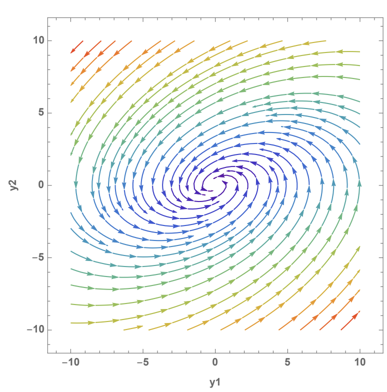
$$\frac{\partial f_1}{\partial y_1} = y_2^2|_{y_1=0,y_2=0} = 0, \quad \frac{\partial f_1}{\partial y_2} = 2y_1y_2 - 2|_{y_1=0,y_2=0} = -2,$$

$$\frac{\partial f_2}{\partial y_1} = 1|_{y_1=0,y_2=0} = 1, \quad \frac{\partial f_2}{\partial y_2} = -1|_{y_1=0,y_2=0} = -1 \quad (4 \ marks).$$

Thus, we have the linearized system as

$$\dot{y_1} = -2y_2, \quad \dot{y_2} = y_1 - y_2, \quad \begin{pmatrix} \dot{y_1} \\ \dot{y_2} \end{pmatrix} = \begin{pmatrix} 0 & -2 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}.$$

The characteristic equation is given by $(0 - \lambda)(-1 - \lambda) + 2 = \lambda^2 + \lambda + 2 = 0$ with the two complex roots of the opposite sign $\lambda_{1,2} = -\frac{1}{2} \pm \frac{\sqrt{7}}{2}i$ (2 marks). As the eigenvalues are complex numbers with a negative real part, this equilibrium corresponds to a stable spiral, where trajectories spiral in towards it over time (2 marks). We could directly sketch its phase portraits as



. (5 marks)

The eigenvector corresponding to $\lambda_1 = -\frac{1}{2} + \frac{\sqrt{7}}{2}i$ can be found from

$$\begin{pmatrix} 0 & -2 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} p_1 \\ q_1 \end{pmatrix} = (-\frac{1}{2} + \frac{\sqrt{7}}{2}i) \begin{pmatrix} p_1 \\ q_1 \end{pmatrix}, \quad \Rightarrow -2q_1 = -\frac{p_1}{2} + \frac{\sqrt{7}p_1}{2}i$$

or equivalently $p_1 = \frac{q_1}{2} + \frac{\sqrt{7}q_1}{2}i$ (2 mark).

so that the eigenvector can be chosen as $\mathbf{u}_1 = \begin{pmatrix} 1 \\ \frac{1}{4} - \frac{\sqrt{7}}{4}i \end{pmatrix}$ by setting $p_1 = 1$(2 mark) (or equivalently $\mathbf{u}_1 = \begin{pmatrix} \frac{1}{2} + \frac{\sqrt{7}}{2}i \\ 1 \end{pmatrix}$ by setting $q_1 = 1$). Similarly, we have the eigenvector for $\lambda_2 = -frac12 - \frac{\sqrt{7}}{2}i$, $\mathbf{u}_2 = \begin{pmatrix} 1 \\ \frac{1}{4} + \frac{\sqrt{7}}{4}i \end{pmatrix}$ or $\mathbf{u}_2 = \begin{pmatrix} \frac{1}{2} - \frac{\sqrt{7}}{2}i \\ 1 \end{pmatrix}$(2 mark).

Note you can also sketch the phase portrait based on the general solution as

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = C_1 e^{(-\frac{1}{2}+\frac{\sqrt{7}}{2}i)t} \begin{pmatrix} 1 \\ \frac{1}{4} - \frac{\sqrt{7}}{4}i \end{pmatrix} + C_2 e^{(-\frac{1}{2}-\frac{\sqrt{7}}{2}i)t} \begin{pmatrix} 1 \\ \frac{1}{4} + \frac{\sqrt{7}}{4}i \end{pmatrix}.$$

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