Queen Mary
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MTH5123 Differential Equations

Revision of pre-request knowledge from Calculus and Algebra **G.Bianconi**

This sheet revises material from Calculus and Algebra: basic differentiation and integration techniques, curve sketching, and solving linear systems; more advanced problems are also included to help you refine your analytical skills from these modules. *Many of the problems are formulated so that you can* **investigate and write**. *You* **should** *work through all of the exercises.* Try checking your computational answers using Mathematica, MATLAB, etc. Selected solutions will be made available on the MTH5132 QMPlus page.

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1. Compute the derivative $f'(x)$ of the following functions:

a)	$f(x) = (x-1)(x^2+1)$	**b)**	$f(x) = 1/(1-x^2)$	**c)**	$f(x) = x/(1+x^2)$
d)	$f(x) = (1-x^4)/(1+x^2)$	**e)**	$f(x) = xe^{-x}$	**f)**	$f(x) = x\sin(x)$
g)	$f(x) = x\cos(x^2)$	**h)**	$f(x) = x\ln\|x\|$	**i)**	$f(x) = 1/(x\ln\|x\|)$
j)	$\sqrt{1+(\sin x)^2}$	**k)**	$\arctan\frac{2x}{1-x^2}$	**l)**	$\ln\tan(x/2)$
m)	$\sinh x\cosh x$	**n)**	$f(x) = (\cos(x)+\sin(x)\tan(x))\cos(x)$		

2. Compute the indefinite integral $\int f(x)\,dx$ of the following functions:

a)	$f(x) = (x-1)(x^2+1)$	**b)**	$f(x) = 1/(1-x^2)$	**c)**	$f(x) = x/(1+x^2)$
d)	$f(x) = (1-x^4)/(1+x^2)$	**e)**	$f(x) = xe^{-x}$	**f)**	$f(x) = x\sin(x)$
g)	$f(x) = x\cos(x^2)$	**h)**	$f(x) = x\ln\|x\|$	**i)**	$f(x) = 1/(x\ln\|x\|)$
m)	$\sinh x\cosh x$	**n)**	$f(x) = (\cos(x)+\sin(x)\tan(x))\cos(x)$		

3. Compute the zeros, the maxima/minima, the limit when x goes to infinity of the following functions: **a)** $f(x) = 2e^{-2x} - e^{-x}$, **b)** $f(x) = e^{-x}(\cos(x)+\sin(x))$

4. Describe and/or sketch the curves representing solutions to the algebraic equations:

$$\textbf{a)}\ x^2+y^2=4, \qquad \textbf{b)}\ (x-1)y=2, \qquad \textbf{c)}\ y^2+3x=0$$

5. Rewrite the system of equations in a matrix form and solve by inversion:

a) $2x+3y=5, \quad 3x+2y=5$

b) $c_1+\frac{1}{2}c_2=1, \quad 2c_1+3c_2=2$

6. The follow exercise requires understanding complex numbers.

a) Find all roots of the polynomial $x^8 - 1$.

b) Describe the nature (real vs. complex, distinct vs. coincident) of the solutions of the equation $x^2(1-x) = x + \lambda x^2$, as a function of the real parameter λ. *Hint: The values $\lambda = -1, 3$ are the interesting ones.*

7. Study the function $|\csc(x) + \cot(x)|$, and its logarithm.

8. Consider the function $f(x) = \ln \frac{x^2}{x-1}$. Determine the domain of f. Show that f is not invertible, and find the two branches of f^{-1}. *Hint: The square root has two branches.*